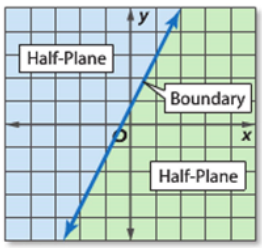
4-6: Graphing Inequalities in Two Variables

**1 Graph Linear Inequalities** The graph of a linear inequality is the set of points that represent all of the possible solutions of that inequality. An equation defines a **boundary**, which divides the coordinate plane into two **half-planes**.




The boundary may or may not be included in the solution. When it is included, the solution is a **closed half-plane**. When not included, the solution is an **open half-plane**.

**KeyConcept Graphing Linear Inequalities**

**Step 1** Graph the boundary. Use a solid line when the inequality contains ≤ or ≥. Use a dashed line when the inequality contains < or >.

**Step 2** Use a test point to determine which half-plane should be shaded.

**Step 3** Shade the half-plane that contains the solution.

"dotted line"

think closed circle ●

think open circle ○

EXERCISE 1 Graph $3x - y < 2$.

First, we need to find our **boundary line**. This is what we put into the graphing calculator. So let's solve $3x - y = 2$ for y:

$3x - y = 2$

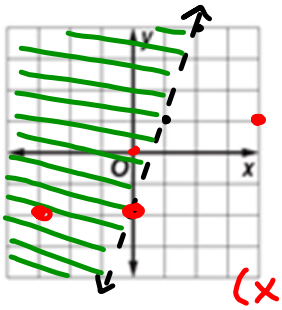
$-3x \quad -3x$

$\frac{-1y}{-1} = \frac{-3x}{-1} + \frac{2}{-1}$

$y = 3x - 2$ boundary line

Now, put that equation into the calculator and find the points we need to graph in the table. Plot those points on the graph. DO NOT DRAW THE LINE YET!

PLEASE SHADE NEATLY




(x, y)

Go back to the original inequality $3x - y < 2$.

Do we have <, >, ≤, or ≥? <

Do we need a SOLID line or DASHED line?

dashed

Now, draw the line accordingly.

Next, select a "test point." (0, 0) usually makes a good test point (unless your boundary goes through the origin, then select something else). Plug your test point into $3x - y < 2$. What happens?

$3(0) - 0 < 2$

$0 - 0 < 2$

$0 < 2$

TRUE

If the result is TRUE, shade the half-plane that contains that point. If the result is FALSE, shade the half-plane that does NOT contain that point.

$3x - y < 2$

Test (x, y) = (−3, −2):

$3(-3) - (-2) < 2$

$-9 + 2 < 2$

$-7 < 2$

TRUE

Test (x, y) = (4, 1):

$3(4) - 1 < 2$

$12 - 1 < 2$

$11 < 2$

FALSE

Test (x, y) = (0, −2):

$3(0) - (-2) < 2$

$0 + 2 < 2$

$2 < 2$

FALSE

EXERCISE 2 Use a graph to solve $3x + 5 < 14$.

First, graph the boundary, which is the related equation. Replace the inequality sign with an equals sign, and solve for x.

Solve $3x + 5 = 14$

$-5 \quad -5$

$3x = 9$

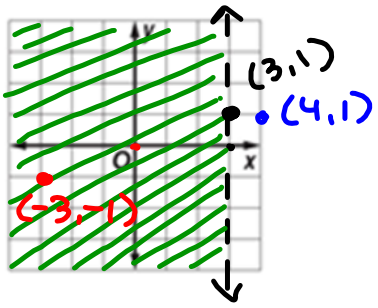
$\frac{3x}{3} = \frac{9}{3}$

$3x+5<14$

dashed

$x = 3$ boundary line

Vertical line




Graph the line of this solution with a SOLID (DASHED) line (circle one).

Why are we using this type of boundary line? because <

Next, choose a test point (remember (0, 0) is usually good). Test this point in the original inequality. Is the result TRUE or FALSE? Shade in the half-plane that corresponds accordingly.

$3x+5<14$

$3(0)+5<14$

$0+5<14$

$5<14$ True

$(0,0)$ → is a solution

Finally, examine your graph and identify the solution: $x < 3$

Test $(-3,-1)$

$3(-3)+5<14$

$-9+5<14$

$-4<14$

True

$3x+5<14$

Test $(4,1)$

$3(4)+5<14$

$12+5<14$

$17<14$

False

Test $(3,1)$

$3(3)+5<14$

$9+5<14$

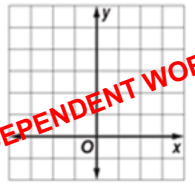
$14<14$

false

TRY SOME ON YOUR OWN:

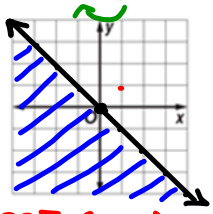
For problems 1-6, graph each inequality.

1. $y < 4$




INDEPENDENT WORK

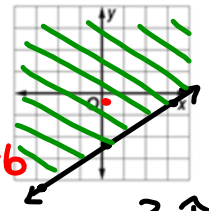
2. $-x \geq y$




$y \leq -x$
solid
boundary line:
$y = -1x + 0$
$y = mx + b$
$m = \frac{-1}{1}$ ↓1 →1 ↑1 ←1
$b = 0$

Test (1,1):
$-1 \geq 1$
FALSE not solution

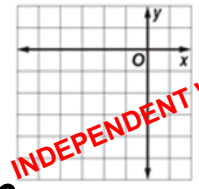
3. $2x - 3y \leq 6$ solid




Test (0,0)
$2(0) - 3(0) \leq 6$
$0 - 0 \leq 6$
$0 \leq 6$
True

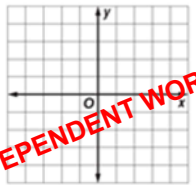
boundary line:
$2x - 3y = 6$
$-2x \downarrow \quad -2x$
$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$
$y = \frac{2}{3}x - 2$
$y = mx + b$
$m = \frac{2}{3}$ ↑2 →3 ↓2 ←3

4. $y < -\frac{1}{2}x - 3$



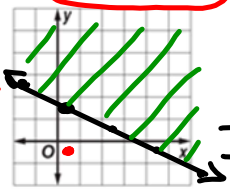

INDEPENDENT WORK

5. $x - y \geq 1$




INDEPENDENT WORK

6. $3x + 6y \geq 12$ solid




x y
Test (0,0)
$3(0) + 6(0) \geq 12$
$0 + 0 \geq 12$
$0 \geq 12$
False

(0,0) is not a solution.

boundary line
$3x + 6y = 12$
$-3x \downarrow \quad -3x$
$\frac{6y}{6} = \frac{-3x}{6} + \frac{12}{6}$
$y = \frac{-1}{2}x + 2$
$b = 2$
$m = \frac{-1}{2}$ ↓1 →2 ↑1 ←2